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Novel universality classes of coupled driven diffusive systems

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Motivated by the phenomenologies of dynamic roughening of strings in random media and magnetohydrodynamics, we examine the universal properties of driven diffusive system with coupled fields. We demonstrate that cross correlations between the fields lead to amplitude ratios and scaling exponents varying continuously with the strength of these cross correlations. The implications of these results for experimentally relevant systems are discussed.

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Recently significant advances have been made in classifying the physics of nonequilibrium systems at long time and length scales into universality classes. It has been shown that standard universality classes in critical dynamics are quite robust to detailed-balance violating perturbations [1]. Novel features are found only for models with conserved order parameter and spatially anisotropic noise correlations. In contrast, truly nonequilibrium dynamic phenomena, whose steady state cannot be described in terms of a Gibbsian distribution, are found to be rather sensitive to all kinds of perturbations. Prominent examples are driven diffusive systems [2] and diffusion-limited reactions [3]. For example, one finds that for the Kardar-Parisi-Zhang (KPZ) equation anisotropic perturbations are relevant in $d>2$ spatial dimensions, leading to rich phenomena that include novel universality classes and the possibility of first-order phase transitions and multicritical behavior [4].

In this paper we study driven nonequilibrium processes described by a set of dynamic variables whose dynamics is given in terms of coupled Langevin equations. Prominent examples include the dynamic roughening of strings moving in random media [5], sedimenting colloidal suspensions [6] and crystals [7], and magnetohydrodynamics (MHD) [8]. Our goal is to investigate and elucidate some of the dramatic effects of symmetries of correlation functions on the universal properties of such systems. We focus on models with two vector fields, $\mathbf{u}(\mathbf{x},t)$ and $\mathbf{b}(\mathbf{x},t)$, as hydrodynamic variables. The quantities of interest are the two autocorrelation functions, $C^u_{ij}(\mathbf{x},t)=\langle u_i(\mathbf{x},t)u_j(\mathbf{0},0)\rangle$ and $C^b_{ij}(\mathbf{x},t)=\langle b_i(\mathbf{x},t)b_j(\mathbf{0},0)\rangle$, and the cross-correlation function $C^\times_{ij}(\mathbf{x},t)=\langle u_i(\mathbf{x},t)b_j(\mathbf{0},0)\rangle$; indices i,j refer to Cartesian coordinates. All these quantities are tensors, whose symmetry properties depend on the model under consideration. We are interested in systems with translational and rotational symmetry, and inversion symmetry such that $\mathbf{u}$ is a polar and $\mathbf{b}$ is an axial vector.

In the first part of the paper, we will consider a one-dimensional Burgers-like model [8] of magnetohydrodynamics and its d-dimensional generalization [10]

$$\frac{\partial \mathbf{u}}{\partial t}+\frac{\lambda_1}{2}\nabla u^2+\frac{\lambda_2}{2}\nabla b^2=\nu\nabla^2\mathbf{u}+\mathbf{f}, \tag{1}$$

$$\frac{\partial \mathbf{b}}{\partial t}+\lambda_3\nabla(\mathbf{u}\cdot\mathbf{b})=\mu\nabla^2\mathbf{b}+\mathbf{g}. \tag{2}$$

Here λ_i are coupling constants, ν and μ are the dissipation coefficients, and $\mathbf{f}$ and $\mathbf{g}$ are external stochastic forcing functions. These equations are simplified versions of the dynamical equations governing the time evolution of the velocity $\mathbf{u}$ and the magnetic field $\mathbf{b}$ in a magnetized fluid (MHD). They are constructed in the same spirit as Burgers equation from the Navier-Stokes equation. In the second part of the paper we will discuss the advection of a passive vector $\mathbf{b}$, where $\lambda_1=\lambda_2=0$. The simplicity of such a model will allow us to explore higher order correlation functions.

For Langevin equations describing processes relaxing towards a thermal equilibrium state the correlation functions for the noise have to obey detailed balance conditions. In nonequilibrium models there are no such restrictions. As a minimal requirement one might ask that the noise terms $\mathbf{f}$ and $\mathbf{g}$ in the Langevin equations obey the same symmetries as the correlation functions for the hydrodynamic fields. Since $\mathbf{u}$ is a polar vector and $\mathbf{b}$ is an axial vector, $\langle u_i(\mathbf{k},t)u_j(-\mathbf{k},0)\rangle,\langle b_i(\mathbf{k},t)b_j(-\mathbf{k},0)\rangle$ are real and even in $\mathbf{k}$, but the cross-correlation function $C^\times_{ij}(\mathbf{k},t)=\langle u_i(\mathbf{k},t)b_j(-\mathbf{k},0)\rangle$ is imaginary and odd in $\mathbf{k}$ [8]. Then, assuming Gaussian distributed conserved noise with zero mean, the noise correlation functions have to be of the following form

$$\langle f_i(\mathbf{k},t)f_j(-\mathbf{k},0)\rangle=2k_ik_jD_u^{(0)}(\mathbf{k})\,\delta(t), \tag{3}$$

$$\langle g_i(\mathbf{k},t)g_j(-\mathbf{k},0)\rangle=2k_ik_jD_b^{(0)}(\mathbf{k})\,\delta(t), \tag{4}$$

$$\langle f_i(\mathbf{k},t)g_j(-\mathbf{k},0)\rangle=2iD_{ij}^{\times(0)}(\mathbf{k})\,\delta(t), \tag{5}$$

where the noise variances $D^{(0)}_{u,b}(\mathbf{k})$ are even and $D^{\times(0)}_{ij}(\mathbf{k})$ is odd in $\mathbf{k}$, respectively. Equations (3) and (4) are invariant under inversion, rotation, and exchange of i with j. We take the noise cross correlation, Eq. (5), to be invariant under inversion, but we allow it to break rotational invariance or symmetry with respect to an interchange of the Cartesian indices i and j.

We are interested in the physics at long time and length scales. Then all the correlation functions $C(\mathbf{x},t)$ are expected to obey scaling relations of the form

$$C(\mathbf{x},t)=x^{2\chi}C(t/x^z). \tag{6}$$

Since we have two independent fields **u** and **b** there could in principle be two different roughness exponents $\chi_{u,b}$. Due to Galilean invariance, however, none of the nonlinearities in the equations of motion renormalize, and one gets $\chi_u=\chi_b=\chi=2-z$ [8,9].

Symmetric cross correlations. If both the fields are irrotational, one can introduce two scalar fields h and ϕ such that $\mathbf{u}=\boldsymbol{\nabla}h$ and $\mathbf{b}=\boldsymbol{\nabla}\phi$; note that ϕ is actually a pseudoscalar. Then Eqs. (1) and (2) become identical to a model of Ertaş and Kardar [5] describing the dynamic roughening of directed lines

$$\frac{\partial h}{\partial t}+\frac{\lambda_1}{2}(\nabla h)^2+\frac{\lambda_2}{2}(\nabla\phi)^2=\nu\nabla^2 h+\eta_h, \tag{7}$$

$$\frac{\partial\phi}{\partial t}+\lambda_3(\boldsymbol{\nabla}h)\cdot(\boldsymbol{\nabla}\phi)=\mu\nabla^2\phi+\eta_\phi, \tag{8}$$

where $\mathbf{f}=\boldsymbol{\nabla}\eta_h$ and $\mathbf{g}=\boldsymbol{\nabla}\eta_\phi$. The cross-correlation function $D_{ij}^{\times(0)}$ is now symmetric in the tensor indices and $\langle h(\mathbf{k},0)\phi(-\mathbf{k},0)\rangle$ is imaginary and odd in **k**. If, in addition, we require rotational invariance, the cross-correlation function would vanish. This is the case considered in Ref. [5]. For a truly nonequilibrium model there is, however, no physical principle which would exclude a finite cross-correlation term *a priori*. Hence we allow for a nonzero $\langle\eta_h(\mathbf{k},0)\eta_\phi(-\mathbf{k},0)\rangle$, which then explicitly breaks rotational invariance, and explore its consequences for the dynamics.

We have determined the roughness exponent χ and the dynamic exponent z employing a lowest order self-consistent mode-coupling scheme and a one-loop dynamic renormalization group calculation. Perturbation theory is formulated in terms of the response and correlation functions for the fields h and ϕ. They are conveniently written in terms of self-energies $\Sigma(k,\omega)$ and generalized kinetic coefficients $D(k,\omega)$. For simplicity we assume that $\nu=\mu$; in MHD this would correspond to a system with magnetic Prandtl number $\mathrm{P}_m=\mu/\nu=1$. Then there is only one response function and it can be written as $G_{h,\phi}^{-1}(\mathbf{k},\omega)=i\omega-\Sigma(\mathbf{k},\omega)$. Then, correlation functions are of the form, $C_\alpha(\mathbf{k},\omega)=2D_\alpha(\mathbf{k},\omega)|G(\mathbf{k},\omega)|^2$ for $\alpha=h,\phi$ and $C_\times(\mathbf{k},\omega)=2iD_\times(\mathbf{k},\omega)|G(\mathbf{k},\omega)|^2$ for the cross-correlation function. In diagrammatic language lowest order mode-coupling theory is equivalent to a self-consistent one loop theory. The ensuing coupled set of integral equations is compatible with the scaling form Eq. (6). In Fourier space the scaling form reads for the self-energy, $\Sigma(k,\omega)=\Gamma k^z\sigma(\omega/k^z)$, and for the generalized kinetic coefficients $D_h(k,\omega)=D_h k^{-d-2\chi}d_h(\omega/k^z)$, $D_\phi(k,\omega)=D_\phi k^{-d-2\chi}d_\phi(\omega/k^z)$, $D_\times(\mathbf{k},\omega)=\mathrm{sgn}(\mathbf{k})D_\times k^{-d-2\chi}d_\times(\omega/k^z)$. To solve this set of coupled integral equations we employ a small χ expansion [11]. This requires matching of the self-energies and correlation functions at $\omega=0$. With the zero-frequency expressions $\Sigma(\mathbf{k},0)=\Gamma k^z$, $D_h(\mathbf{k},0)=D_h k^{-2\chi-d}$, $D_\phi(\mathbf{k},0)=D_\phi k^{-2\chi-d}$, one finds for the one-loop self-energy (we take $\lambda_1=\lambda_2=\lambda_3=\lambda$ without any loss of generality)

$$\frac{\Gamma^2}{D_h\lambda^2}=\frac{S_d}{(2\pi)^d}\frac{1}{2d}\left(1+\frac{D_\phi}{D_h}\right), \tag{9}$$

and for the one-loop correlation functions,

$$\frac{\Gamma^2}{D_h\lambda^2}=\frac{1}{4}\frac{S_d}{(2\pi)^d}\frac{1}{d-2+3\chi}\left[1+\left(\frac{D_\phi}{D_h}\right)^2+2\left(\frac{D_\times}{D_h}\right)^2\right],$$

$$\frac{\Gamma^2}{D_\phi\lambda^2}=\frac{1}{2}\frac{S_d}{(2\pi)^d}\frac{1}{d-2+3\chi}\left[\frac{D_h}{D_\phi}-\left(\frac{D_\times}{D_\phi}\right)^2\right]. \tag{10}$$

Here S_d is the surface of a d-dimensional unit sphere. From Eqs. (10) we find

$$\left(\frac{D_\phi}{D_h}\right)^2+2N\left(\frac{D_h}{D_\phi}+1\right)-1=0, \tag{11}$$

where $N\equiv(D_\times/D_h)^2$ defines an amplitude ratio. In the Eq. (11), the domain of N is determined by the range of real values for D_ϕ/D_h starting from 1 (for $N=0$). Thus for small N we can expand around 0 and look for solutions of the form $D_\phi/D_h=1+aN$, such that for $N=0$ we recover $D_h=D_\phi$ (the result of Ref. [5]). We obtain $a=-2$, i.e.,

$$D_\phi/D_h=1-2N, \tag{12}$$

implying that within this approximate calculation N cannot exceed 1/2, i.e., $D_\times\leqslant D_h/2$. An important consequence of this calculation is that the amplitude ratio D_ϕ/D_h is no longer fixed to 1 but can vary continuously with the strength of the noise cross-correlation amplitude $D_\times$. These results are confirmed by a one-loop renormalization group (RG) for the strong coupling fixed point in $d=1$. In addition, Eq. (12) is valid at the roughening transitions to lowest order in a $d=2+\epsilon$ expansion.

In contrast, the scaling exponents χ and z are not affected by the presence of cross correlations. We get $\chi=\frac{1}{2}$ and $z=\frac{3}{2}$ in $d=1$ dimensions, $\chi=-O(\epsilon)^2$ and $z=2+O(\epsilon)^2$ at the roughening transitions in a $d=2+\epsilon$ expansion, and $\chi=2-(d/2)$ and $z=\frac{4}{3}+(d/6)$ for the strong coupling phase. Note that the values for the strong coupling exponents are obtained within Bhattacharjee's [11] small-χ expansion, as described above. There is still an ongoing debate whether those values for the exponents actually correspond to the KPZ strong coupling case (see Ref. [12] for a discussion).

We have verified our analytical results in $d=1$ by numerical simulations of both a coupled lattice model with cross correlations, and direct numerical simulations of the model Eqs. (1) and (2). Our numerical results explicitly demonstrate the dependence of the amplitude ratio on the cross-correlation function amplitude. Details will be published elsewhere [13].

Antisymmetric cross correlations. In the preceding paragraph we have restricted ourselves to irrotational fields. If the vector fields $\mathbf{a}=\mathbf{u},\mathbf{b}$ have the form $\mathbf{a}=\boldsymbol{\nabla}\times\mathbf{V}_a+\boldsymbol{\nabla}S_a$, with vectors $\mathbf{V}_a$ being cross correlated but scalars S_a uncorrelated then the variance $D_{ij}^\times$ satisfies $D_{ij}^\times(\mathbf{k})=-D_{ij}^\times(-\mathbf{k})=D_{ji}^\times(-\mathbf{k})=-[D_{ij}^\times(\mathbf{k})]^*$. This is the antisymmetric part of the cross correlations. The noise strength $\tilde{D}_\times$ is defined by

$D^{\times}_{ij}(\mathbf{k})D^{\times}_{ji}(-\mathbf{k})=\tilde{D}^2_{\times}k^4$. In the scaling limit, the self-energy reads $\Sigma(k,\omega)=\Gamma k^z\sigma(\omega/k^z)$, the correlation functions are $C^u_{ij}(k,\omega)=k_ik_jD_uk^{-d-2\chi}d_u(\omega/k^z)$, $C^b_{ij}(k,\omega)=k_ik_jD_bk^{-d-2\chi}d_b(\omega/k^z)$, and the antisymmetric part of the cross correlation function reads $C^a_{ij}(k,\omega)=D^a_{ij}(\mathbf{k})k^{-2\chi-z-d}$.

Following methods used for the symmetric cross correlations, we obtain the analogues of Eqs. (9) and (10)

$$\frac{\Gamma^2}{D_u\lambda^2}=\frac{S_d}{(2\pi)^d}\frac{1}{2d}\left(1+\frac{D_b}{D_u}\right), \tag{13}$$

$$\frac{\Gamma^2}{D_u\lambda^2}=\frac{1}{4}\frac{S_d}{(2\pi)^d}\frac{1}{d-2+3\chi}\left[1+\left(\frac{D_b}{D_u}\right)^2+2\left(\frac{\tilde{D}_\times}{D_u}\right)^2\right],$$

$$\frac{\Gamma^2}{D_b\lambda^2}=\frac{1}{2}\frac{S_d}{(2\pi)^d}\frac{1}{d-2+3\chi}\left[\frac{D_u}{D_b}+\left(\frac{\tilde{D}_\times}{D_b}\right)^2\right]. \tag{14}$$

Equations (13) and (14) give $D_u/D_b=1$ at the fixed point for arbitrary values of $\tilde{N}=(\tilde{D}_\times/D_h)^2$. Hence no restrictions on $\tilde{N}$ arises from that. In contrast to the effects of the symmetric cross correlations, the exponents now depend continuously on $\tilde{N}$. To leading order, we get

$$\chi=\frac{2}{3}-\frac{d}{6}+\frac{\tilde{N}d}{6}, \quad z=\frac{4}{3}+\frac{d}{6}-\frac{\tilde{N}d}{6}. \tag{15}$$

These exponents presumably describe the rough phase above $d>2$, with the same caveats as above [12]. With increasing $\tilde{D}_\times$ the exponent χ also grows (and z decreases). Obviously this cannot happen indefinitely. We estimate the upper limit of $\tilde{N}$ in the following way: Note that the Eqs. (1) and (2) along with the prescribed noise correlations (i.e., equivalently the dynamic generating functional) are of conservation law form, i.e., they vanish as $\mathbf{k}\to\mathbf{0}$. Thus there is no information of any infrared cutoff in the dynamic generating functional. Moreover, we know the solutions of the equations *exactly* if we drop the nonlinear terms (and hence, the exponents: $\chi=1-d/2, z=2$). Hence physically relevant quantities like the total kinetic and magnetic energies, $\int_k\langle\mathbf{u}(\mathbf{k},t)\mathbf{u}(-\mathbf{k},t)\rangle$ and $\int_k\langle\mathbf{b}(\mathbf{k},t)\mathbf{b}(-\mathbf{k},t)\rangle$, remain *finite* as the system size diverges, and are thus independent of the system size. Since the nonlinear terms are of the conservation law form, inclusion of them *cannot* bring a system size dependence on the values of the total kinetic and magnetic energies. However, if χ continues to increase with $\tilde{D}_\times$ at some stage these energies would start to depend on the system size which is unphysical [14]. So we have to restrict $\tilde{N}$ to values smaller than the maximum value for which these energy integrals are just system size independent: This gives $\tilde{N}^{\max}=(2/d)(d/2+1)$. Note that the limits on N and $\tilde{N}$ impose consistency conditions on the amplitudes of the measured correlation functions but not on the bare noise correlators.

Antisymmetric cross correlations stabilize the short-range fixed point with respect to perturbations by long-range noise with correlations $\propto k^{-y}, y>0$. This can easily be seen; in presence of noise correlations sufficiently singular in the infrared limit, i.e., large enough y, the dynamic exponent is *exactly* given by [12,15] $z_{\mathrm{lr}}=(2+d/3)-(y/3)$. The short-range fixed point remains stable as long as $z_{\mathrm{sr}}<z_{\mathrm{lr}}$ which gives $y<-2+(1+\tilde{N})d/2$. Hence we conclude that in presence of antisymmetric cross-correlations long-range noise must be *more singular* for the short-range noise fixed point to lose its stability or in other words, antisymmetric cross correlations increases the stability of the short-range noise fixed point with respect to perturbations from long-range noises.

We have seen that the amplitudes of the cross-correlation function play a quite crucial role in determining the long wavelength properties of the system. In our analysis we used only short-range noise, which is enough to elucidate the basic points. However, a Langevin description of many systems often requires a noise term with correlations becoming singular in the long wavelength limit, such as fully developed MHD [8]. These systems are typically characterized by a set of anomalous exponents for higher order correlation functions. Below we give an illustrative example to highlight the effects of symmetries on the anomalous scaling exponents of higher order correlation functions in the passive vector limit where the velocity field $\mathbf{u}$ is assumed to obey a Gaussian distribution [instead of Eq. (1)] with a variance $\langle u_i(\mathbf{k},t)u_j\times(-\mathbf{k},0)\rangle=[2D\delta(t)/(k^2+M^2)^{d/2+\epsilon/2}][\alpha P_{ij}+Q_{ij}]$ where $0\leqslant\epsilon\leqslant 2$, which makes the model analytically tractable. As before, the magnetic field $\mathbf{b}$ is governed by Eq. (2). The tensor P_{ij} is the transverse projection operator, Q_{ij} is the longitudinal projection operator. The parameter $\alpha>0$ determines the extent of incompressibility of the $\mathbf{u}$ field. Thus in this problem α appears as a tuning parameter in the multiplicative noise, very much like N and $\tilde{N}$ appeared in our previous results. By following a field-theoretic dynamic renormalization group procedure in conjunction with operator product expansion [16], we calculate the scaling exponents of the structure functions $S_n(r)=\langle[\theta(\mathbf{x}+\mathbf{r})-\theta(\mathbf{x})]^{2n}\rangle\sim r^{\zeta_n}, \mathbf{b}=\nabla\theta$. Within a one-loop approximation we find

$$\zeta_n=2n-\frac{n\epsilon d}{d\alpha+1-\alpha}\left[\alpha+\frac{1-\alpha}{d}+2(n-1)\left\{\frac{\alpha}{d}+\frac{3(1-\alpha)}{d(d+2)}\right\}\right]. \tag{16}$$

This clearly demonstrates that even for the linear problem there is an continuous dependence of the scaling exponents on the parameter α, characterizing the extent to which the velocity field is compressible. We expect this to hold also for the nonlinear problem, whose analysis is significantly more complicated.

Let us now review our results in the context of some physically relevant systems. Our results are relevant for a wide class of nonequilibrium systems. In MHD turbulence the cross-correlation function $\langle u_i(\mathbf{k},t)b_j(-\mathbf{k},t)\rangle$ is, in general, nonzero [17], and as before, is odd and imaginary in $\mathbf{k}$. Similar calculations as here for MHD show that two dimensionless numbers, the magnetic Prandtl number P_m and the ratio of the magnetic to the kinetic energy, are nonuniversal; they are functions of both the symmetric and antisymmetric part of the cross-correlation amplitudes. Another system of interest is the dynamics of a drifting polymer through a solution [18]. Here the hydrodynamic degrees of freedom are the transverse and longitudinal displacements with respect to the mean position. Dynamic light scattering experiments can be performed to investigate the effects of cross correlations discussed here. Our results are significant also for coupled growth of nonequilibrium surfaces [20], and sedimenting lattices [7,21].

In summary, we have demonstrated that cross correlations between two vector fields can drastically alter their asymptotic statics and dynamics at long length and time scales. The symmetric and antisymmetric part of the noise cross-correlation function have different effects. The symmetric part leaves the scaling exponents unaffected but yields amplitude ratios of the various correlation functions, which continuously depend on the amplitude of the noise cross correlation [see Eq. (12)]. In contrast, the asymmetric part leaves the amplitude ratios unaffected, but leads to continuously varying exponents [see Eq. (15)]. In both cases the continuous variation with the noise amplitude of the cross correlations is not arbitrary but constrained by scaling relations [see Eqs. (12) and (15)], a feature, present also in our results on the multiplicative noise driven linear system. We have shown this using renormalization group methods and mode-coupling theory, confirmed by some preliminary simulations [13]. Recently, Drossel and Kardar [19] have studied a set of coupled Langevin equations describing the interplay between phase ordering dynamics in the bulk and roughening dynamics of the interface of binary films. They find a similar continuous variation of the dynamical exponent with the coupling strength of the bulk and surface fields. Nonperturbative analysis or numerical simulations may be necessary to resolve the questions about the rough phase more satisfactorily. In the light of our results it might also be interesting to examine the effects of cross correlations on the multiscaling properties of MHD in experiments and/or numerical simulations.

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